M10/5/MATHL/HP3/ENG/TZ0/SP

22107210

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MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Thursday 20 May 2010 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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4 pages

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 12]*

Anna cycles to her new school. She records the times taken for the first ten days with the following results (in minutes).

12.4 13.7 12.5 13.4 13.8 12.3 14.0 12.8 12.6 13.5

Assume that these times are a random sample from the $\mathrm{N}(\mu, \sigma^2)$ distribution.

(a) Determine unbiased estimates for μ and σ^2. *[2 marks]*

(b) Calculate a 95 % confidence interval for μ. *[3 marks]*

(c) Before Anna calculated the confidence interval she thought that the value of μ would be 12.5. In order to check this, she sets up the null hypothesis $\mathrm{H}_0 : \mu = 12.5$.

(i) Use the above data to calculate the value of an appropriate test statistic. Find the corresponding p-value using a two-tailed test.

(ii) Interpret your p-value at the 1 % level of significance, justifying your conclusion. *[7 marks]*

2. *[Maximum mark: 10]*

The random variable X has a Poisson distribution with mean μ. The value of μ is known to be either 1 or 2 so the following hypotheses are set up.

$$\mathrm{H}_0 : \mu = 1;\ \mathrm{H}_1 : \mu = 2$$

A random sample $x_1, x_2, \ldots, x_{10}$ of 10 observations is taken from the distribution of X and the following critical region is defined.

$$\sum_{i=1}^{10} x_i \ge 15$$

Determine the probability of

(a) a Type I error; *[5 marks]*

(b) a Type II error. *[5 marks]*

3. *[Maximum mark: 13]*

The random variable X is assumed to have probability density function f, where

$$f(x) = \begin{cases} \dfrac{x}{18}, & 0 \le x \le 6 \\ 0, & \text{otherwise}. \end{cases}$$

(a) Show that if the assumption is correct, then

$$\text{P}(a \le X \le b) = \frac{b^2 - a^2}{36}, \text{ for } 0 \le a \le b \le 6.$$ *[3 marks]*

(b) A random sample of 180 values of X was obtained and the following table produced.

Interval	$[0, 1[$	$[1, 2[$	$[2, 3[$	$[3, 4[$	$[4, 5[$	$[5, 6]$
Frequency	8	18	24	37	44	49

Stating appropriate hypotheses, test the assumption at the 5 % level of significance using a χ^2 test. *[10 marks]*

4. *[Maximum mark: 8]*

A shop sells apples, pears and peaches. The weights, in grams, of these three types of fruit may be assumed to be normally distributed with means and standard deviations as given in the following table.

Fruit	**Mean**	**Standard Deviation**
Apples	115	5
Pears	110	4
Peaches	105	3

Alan buys 1 apple and 1 pear while Brian buys 1 peach. Calculate the probability that the combined weight of Alan's apple and pear is greater than twice the weight of Brian's peach.

5. *[Maximum mark: 17]*

(a) A bag contains 20 coloured balls of which 12 are red and 8 are blue. A random sample of 6 of these balls is selected without replacement. Calculate the mean and the variance of the number of red balls in the sample. *[7 marks]*

(b) The random variable X has the negative binomial distribution $\text{NB}(5,\ p)$, where $p < 0.5$, and $\text{P}(X = 10) = 0.05$. By first finding the value of p, find the value of $\text{P}(X = 11)$. *[10 marks]*